

# Baryon masses in a chiral expansion with meson-baryon form factors

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## Abstract

The chiral expansion of the one-loop corrections to baryon masses is examined in a generic meson-cloud model with meson-baryon form factors. For pion loops, the expansion is rapidly convergent and at fourth order in  $m_\pi$  accurately reproduces the full integral. In contrast, the expansion is found to converge very slowly for kaon loops, raising questions about the usefulness of chiral expansions for kaon-baryon physics. Despite the importance of high-order terms, relations like that of Gell-Mann and Okubo are well satisfied by the baryon masses calculated with the full integral. The pion cloud makes a significant contribution to the  $\pi N$  sigma commutator, while kaon cloud gives a very small strangeness content in the nucleon.

## I. INTRODUCTION

The cloud of virtual mesons that surrounds any baryon contributes to the mass and other properties of that particle. This can significantly change these properties compared to expectations based on simple quark models in which the baryon is described as a three-quark state. Of particular importance in this context are the pseudoscalar mesons, pions and kaons, since they form the longest-ranged components of that cloud.

These lightest mesons are approximate Goldstone bosons, whose masses arise from the breaking of chiral symmetry by the current masses of the quarks. Their contributions can thus be systematically expanded in powers of the current-quark masses using the techniques of chiral perturbation theory (ChPT) [1–6]. Although this has been applied to meson-baryon interactions with Dirac nucleons [7], the complications introduced by the finite baryon mass mean that most practical applications are based on heavy-fermion effective field theory [8]. In this the baryons are treated as very heavy and the effective action is expanded in powers of the baryon momenta around the nonrelativistic limit [9]. All ChPT approaches are based on nonrenormalisable effective Lagrangians; hence, as the expansion is taken to higher orders, more and more counterterms are introduced with coefficients that need to be determined.

The lowest-order approximations to heavy-baryon ChPT have much in common with the cloudy bag model [10,11]: static or slow-moving baryons perturbatively dressed with a meson cloud. The cloudy bag model and similar chiral versions of the colour-dielectric soliton model [12–15] are able to give very good descriptions of low-energy baryon properties. Even if they cannot be extended to a complete and rigorous chiral expansion, these models can also provide useful estimates of the coefficients of some of the higher-order counterterms that will be present in such an expansion. In particular these models give rise to form factors at the meson-baryon vertices that regulate the loop integrals involved in calculating the mesonic dressing of baryons.

Within the framework of a cloudy-bag approach, we have examined the chiral expansion of the self-energies of octet and decuplet baryons including one-loop contributions from

the octet of pseudoscalar mesons (pions, kaons and  $\eta$ ). To look for general features of the approach we have used a generic form factor instead of one calculated from the quark wave functions of some specific bag or soliton model. We find that, for reasonable choices of form factor, the chiral expansion of pion loops converges rapidly. The results indicate that inclusion of terms up to fourth order in  $m_\pi$  are sufficient to accurately reproduce the full integral. However the kaon loops converge much more slowly and terms up to at least seventh order in  $m_K$  being needed. The same pattern of an alternating series with very slow convergence is also found in the work of Borasoy and Meissner [16], who calculated the nucleon mass up to fourth order in full SU(3) ChPT. Our results indicate that such calculations are likely to have to be extended to at least seventh order, raising serious doubts about their feasibility. In contrast ChPT calculations with nucleons (and  $\Delta$ 's, see below) and virtual pions are likely to be well converged at the order currently achieved.

As in the cloudy-bag model [11], we have explicitly included the decuplet of spin- $\frac{3}{2}$  baryons in our calculations. These can play a significant role in the mesonic dressing of the octet baryons because the octet-decuplet splitting is comparable in magnitude to the pion mass. Indeed Jenkins and Manohar [17] have argued that decuplet baryon fields should be included in ChPT. This has been applied to calculations of baryon masses [18,19] and  $\sigma$ -commutators [20]. However as has been pointed in Refs. [21,22] inclusion of the decuplet leads to new, unknown counterterms at fourth order in ChPT.

The baryon decuplet plays a particularly important role in the limit where the number of colours  $N_c \rightarrow \infty$ . It can then be thought of as the first excited level of a rotational band based on a hedgehog intrinsic state, as in the Skyrme [23,24] and NJL soliton models [25]. In that limit the octet-decuplet splitting vanishes as  $1/N_c$  and so the chiral expansion must be modified [26]. The large- $N_c$  limit also leads to various relations amongst baryon masses, which are independent of the details of baryon structure [27–29].

Various baryon mass relations, normally regarded as tests of first-order perturbation theory in the SU(3)-breaking [30], are found to be very well satisfied by the full baryon-energies calculated with form factors. Low-order terms in the chiral expansion would suggest

much larger violations of these relations, but these are not reliable because of the very slow convergence of the expansion. In fact we find small violations of the GMO relation, similar to those actually observed. Thus the success of the GMO relation cannot be taken to imply the higher-order terms in the chiral expansion are small. Another illustration of this is the very small strange-quark content in the proton that we find in this approach.

Finally, the dressing of baryons with pseudoscalar mesons on its own does not give a very good description of the full octet and decuplet spectrum, as noted by McGovern [13]. We have therefore examined the effects of symmetry-breaking terms and couplings beyond those of simple SU(6) quark-model wave functions. Such effects can arise when, for example, gluon exchange forces are included in these models [13,14]. Our general conclusions are unaffected by the inclusion of such terms.

## II. CHIRAL EXPANSION OF SELF-ENERGIES

In a generic bag or soliton model, we take the bare mass of an octet or decuplet baryon to be of the form

$$M_A^{(0)} = M_0 + N_s \epsilon_s + \delta, \quad (2.1)$$

where  $N_s$  is the number of strange quarks present in the baryon,  $\epsilon_s$  the additional energy associated with a strange quark,  $\delta$  is the bare octet-decuplet mass splitting for a decuplet baryon and zero for an octet one. (In the present work we do not consider isospin-breaking contributions to the energy.) The mass of a dressed baryon can be written as

$$M_A = M_A^{(0)} + \Sigma_A, \quad (2.2)$$

where  $\Sigma_A$  is the self-energy of the baryon arising from its meson cloud. At one-loop level in the cloudy-bag approach [11] the self-energy of a baryon A is a sum of terms of the form

$$\Sigma_{AB\nu} = -\frac{3}{4\pi^2} \left( \frac{g_{\pi NN}}{2M_N} \right)^2 \frac{(f^{AB\nu})^2}{25} \int_0^\infty \frac{k^4 \mu^2(k) dk}{\omega_\nu(k) (\omega_\nu(k) + M_B^{(0)} - M_A^{(0)})}, \quad (2.3)$$

where  $g_{\pi NN}$  is the pion nucleon coupling constant,  $\mu(k)$  is the normalised meson-baryon form factor, and the energy of a meson of momentum  $k$  and mass  $m_\nu$  is

$$\omega_\nu(k) = \sqrt{k^2 + m_\nu^2}. \quad (2.4)$$

The factor  $f^{AB\nu}$  is the coupling coefficient of baryon A to an intermediate state consisting of baryon B and meson  $\nu$ . Values of these are listed in Table I, which generalises the similar table in [31] by including kaon and  $\eta$  couplings [32] and allowing for an  $F/D$  ratio other than the 2/3 of an SU(6) quark model. More generally, when one goes beyond an SU(6) quark model, there should be four independent coupling constants, the  $F$  and  $D$  octet-octet ones, an octet-decuplet one and a decuplet-decuplet one.

The results presented here are all obtained using a Gaussian form factor,

$$\mu(k) = \exp(-k^2/M^2), \quad (2.5)$$

where the form-factor mass  $M = 660$  GeV has been fit to the pion-nucleon form factor in the colour-dielectric model [33]. We have also looked at other form factors, for example the monopole form  $1/(k^2 + M^2)$ , with form-factor masses in the region of 1 GeV. These do not alter the qualitative behaviour of our results.

We take the squares of the meson masses to be linearly related to the current quark masses (again ignoring isospin breaking):

$$m_\pi^2 = 2B_0\overline{m}, \quad (2.6)$$

$$m_K^2 = B_0(m_s + \overline{m}), \quad (2.7)$$

$$m_\eta^2 = B_0\frac{2}{3}(2m_s + \overline{m}), \quad (2.8)$$

where  $\overline{m}$  is the average of the up- and down-quark current masses. The chiral expansion of the self-energy in terms of the current masses is then equivalent to one in terms of the meson masses.

The self-energy diagrams contain nonanalytic dependences on the meson masses  $m_\nu$  [2,3] and so the chiral expansion is not a straightforward power series. These nonanalytic pieces

correspond to infrared divergences for some of the derivatives of the self energy in the chiral limit where the mesons are massless. To illustrate how such an expansion can be made, we consider first a simplified version of Eq. (2.3) in which the bare baryon splittings are ignored. This corresponds to the part of the self-energy of a nucleon arising from virtual  $\pi N$  states. The denominator in the integrand is then just  $k^2 + m_\nu^2$ .

To extract its nonanalytic parts, we first break the integral at some arbitrary momentum  $\Lambda$  into high- and low-momentum regions. We then take the first  $N$  terms of the expansion of the squared form factor in powers of  $k^2$ ,

$$\mu(k)^2 = \sum_{n=0}^{\infty} d_n k^{2n}, \quad (2.9)$$

and integrate these analytically (in practice using Mathematica [34]). This piece,

$$\Sigma_{AB\nu}^{(1)} = \frac{3}{4\pi^2} \left( \frac{g_{\pi NN}}{2M_N} \right)^2 \frac{(f^{AB})^2}{25} \sum_{n=0}^N d_n \int_0^\Lambda \frac{k^{4+2n} dk}{k^2 + m_\nu^2}, \quad (2.10)$$

contains all nonanalytic dependence on  $m_\nu$  up to order  $m_\nu^{2(N+1)}$ . The first such term is of order  $m_\nu^3$ , with a coefficient that can be checked against the standard ChPT result [2]. The first logarithmic dependence on  $m_\nu$  appears at order  $m_\nu^4 \ln m_\nu$ .

The remaining low-momentum contribution,

$$\Sigma_{AB\nu}^{(2)} = \frac{3}{4\pi^2} \left( \frac{g_{\pi NN}}{2M_N} \right)^2 \frac{(f^{AB})^2}{25} \int_0^\Lambda \frac{k^4 (\mu^2(k) - \sum_{n=0}^N d_n k^{2n}) dk}{k^2 + m_\nu^2}, \quad (2.11)$$

can be expanded as a power series in  $m_\nu$  to order  $m_\nu^{2(N+1)}$  without problems. The coefficients in this series involve integrals that must be evaluated numerically.

Finally the high-momentum part of the integral

$$\Sigma_{AB\nu}^{(3)} = \frac{3}{4\pi^2} \left( \frac{g_{\pi NN}}{2M_N} \right)^2 \frac{(f^{AB})^2}{25} \int_\Lambda^\infty \frac{k^4 \mu^2(k) dk}{k^2 + m_\nu^2} \quad (2.12)$$

can be safely expanded as a power series in  $m_\nu$  since it is free of any infrared divergences. Again the coefficients are evaluated numerically. When these three parts of  $\Sigma_{AB\nu}$  are combined we find that, for our numerical integration, the sum is independent of  $\Lambda$  over the range 200 to 800 MeV. This provides a useful check on both our analytic and numerical calculations.

In Fig. 1 we show the self-energy in this case as a function of the meson mass, along with the results of truncating the expansion at orders up  $m_\nu^7$ . The convergence is very rapid for masses in the region of  $m_\pi$ , with terms up to order  $m_\nu^4$  accurately reproducing the full integral. However for masses around  $m_K$  the convergence is much slower, with terms up to at least order  $m^7$  being needed. The alternating signs of the terms in the series are similar to what is seen in SU(3) ChPT calculations up  $m_\nu^4$  [16]. Those calculations show little evidence of convergence up to that order.

The same techniques can be applied, with a little more effort, to the integral with the full denominator in Eq. (2.3). In making the chiral expansion we use the linear relations between the meson masses and the current masses Eqs. (2.6–2.8) and take the extra energy of a strange quark,  $\epsilon_s$  in Eq. (2.1), to be of order  $m_s$ , and write it in the form

$$\epsilon_s = am_K^2. \quad (2.13)$$

We also include the baryon decuplet in our calculations, with bare energy splitting  $\delta$ . This splitting should remain finite in the chiral limit and so is formally of chiral order  $m^0$ . Nonetheless it is numerically comparable to the pion and kaon masses in size. An expansion in powers of those masses for fixed  $\delta$  would be very poorly convergent. We have therefore chosen to make a simultaneous expansion of the self-energy in terms of the meson masses  $m_\nu$  and the splitting  $\delta$ , keeping terms to all orders in  $m_\nu/\delta$  [35,36,22]. In order to do this, we treat the bare octet-decuplet splitting as if it were proportional to the pion or kaon mass, writing it as

$$\delta = bm_\nu. \quad (2.14)$$

The expansion of the full integral is made as described above, but with the substitution

$$\frac{1}{(k^2 + m_\nu^2)} \rightarrow \frac{1}{\omega(\omega + am_\nu^2 + bm_\nu)} \quad (2.15)$$

in the integrals of Eqs. (2.10–2.12). Some further care is needed if the splitting is larger than the meson mass,  $|b| > 1$ , which is the case for pion. Then the contribution to the self-energy

of the decuplet baryon arising from a pion-octet-baryon intermediate state has an imaginary part, reflecting the instability of that particle. The corresponding pion-decuplet-baryon contribution to the self-energy of the octet baryon is of course purely real. Nonetheless its functional form differs from that for smaller splittings. This can be seen most easily by considering the expressions in the case  $a = 0$ . Then the integrals contain

$$\frac{1}{\omega(\omega + bm_\nu)} = \frac{\omega + bm_\nu}{(k^2 + (1 - b^2)m_\nu^2)}. \quad (2.16)$$

For  $|b| < 1$  the integrals corresponding to the two terms in the numerator of this expression give rise to nonanalytic terms with a logarithmic dependence on  $m_\nu$ . For  $|b| > 1$ , there are no such logarithmic terms; instead the poles of the integrands mean that both integrals develop imaginary parts. In the octet case,  $b$  is negative and these imaginary parts cancel exactly leaving a real baryon mass. For any decuplet state except the  $\Omega$ , the imaginary parts of the pion loop terms survive to leave a complex mass, reflecting the unstable nature of these states.

The dependence of the self-energy on the meson mass in this case is shown in Fig. 2. A bare octet-decuplet splitting of 300 MeV has been used. Also shown are the results of a combined expansion in  $m$  and  $\delta$ . For small  $m$  the dependence on  $\delta$  controls the convergence of the expansion but again keeping terms up to fourth order gives good accuracy in the region of  $m_\pi$ . The convergence of the expansion is otherwise similar to that in Fig. 1.

### III. BEYOND THE SU(6) QUARK MODEL

When baryon energies are evaluated as described in the previous section using the bare splittings and meson-baryon couplings of an SU(6) quark model, the results do not give a very good description of the observed spectrum [13]. For example, if the energy  $\epsilon_s$  of a strange quark is chosen to reproduce the overall splitting of the baryon octet, then the  $\Sigma$ - $\Lambda$  splitting is much too small. Alternatively, if the  $\Sigma$ - $\Lambda$  splitting is reproduced, then the  $\Xi$ - $N$  is not. This is illustrated by the first line of Table 2. We have therefore examined the effect of more general coupling terms on our chiral expansion.



Such terms are routinely included in effective chiral Lagrangians for meson-baryon physics (see, for example, [18]). In a general chiral Lagrangian, the leading-order couplings of mesons to octet baryons are of the form

$$\mathcal{L}_{mBB} = 2D\text{Tr}\bar{B}\gamma^\mu\gamma_5\{A_\mu, B\} + 2F\text{Tr}\bar{B}\gamma^\mu\gamma_5[A_\mu, B], \quad (3.1)$$

where  $B$  denotes the octet baryon fields (expressed in  $3 \times 3$  matrix form) and  $A_\mu$  the axial current formed out the meson fields and their derivatives. For a fuller definition of terms see [18]. The pion-nucleon coupling is related by the Goldberger-Treiman relation to the axial coupling of the nucleon  $g_A$ , which in turn is given by

$$g_A = F + D. \quad (3.2)$$

An SU(6) quark model would require a ratio  $F/D = 2/3$  but a somewhat smaller value  $F/D \simeq 0.57\text{--}0.58$  [37] is deduced from data on semileptonic decays of hyperons. We have therefore examined the dependence of our results on  $F/D$ . As noted below, we find that small changes when  $F/D$  is varied over a realistic range. We have therefore not considered the most general possible SU(3)-symmetric couplings which would involve independent octet-decuplet and decuplet-decuplet coupling constants.

Of more importance is the replacement of the term  $N_s\epsilon_s$  in the bare baryon masses of Eq. (2.1) by a more general symmetry-breaking term. In the case of the baryon octet, two SU(3)-breaking terms of octet form are possible:

$$\mathcal{L}_{\chi SB} = b_F\text{Tr}\bar{B}\{(\xi^\dagger M\xi^\dagger + \xi M\xi), B\} + b_D\text{Tr}\bar{B}[(\xi^\dagger M\xi^\dagger + \xi M\xi), B] \quad (3.3)$$

where  $\xi$  is an SU(3) matrix constructed out of meson fields and  $M$  is the meson mass matrix (again see [18] for a complete definition). A splitting proportional to strangeness, as in Eq. (2.1), is obtained if only a  $b_F$  term is included. The other term does not arise in simple SU(6) quark models, but can appear once gluon-exchange effects are included. The strength of the splitting within the decuplet is also treated as an adjustable parameter,  $b_{10}$ .

In Table 2, we show the results for the baryon masses calculated using the full integrals, with  $F/D = 0.58$  and the other parameters chosen to fit the  $\Xi$ , average of  $\Sigma$  and  $\Lambda$ ,  $\Delta$  and  $\Xi^*$

masses. The corresponding values of the parameters are:  $b_F m_K^2 = 87.1$  MeV,  $b_D m_K^2 = 13.5$  MeV,  $b_{10} m_K^2 = 129.0$  MeV and  $\delta = 318.0$  MeV. An overall constant has been added to bring the nucleon mass up to its observed value. A very good description of all eight masses is obtained (with five adjustable parameters). The results are not sensitive to the  $F/D$  ratio: if  $2/3$  is used then the best fit value of  $b_D m_K^2$  is changed to 16.7 MeV and the other parameters are shifted by less than 1 MeV.

If one were to expand these results to first-order in the symmetry breaking terms, one would get very different values for these masses. Nonetheless, despite the importance of higher-order terms, the full masses continue to satisfy relations that are often assumed to test the octet nature of the  $SU(3)$  breaking terms in the baryon energies. For example, the GMO relation among the octet baryon masses [30] states that the combination

$$\Delta_{GMO} = \frac{3}{4}M_\Lambda + \frac{1}{4}M_\Sigma - \frac{1}{2}M_N - \frac{1}{2}M_\Xi \quad (3.4)$$

should vanish if the  $SU(3)$  breaking is purely octet in form. Similarly for the decuplet one can construct two equal spacing rules (DES I and II in the notation of [27]):

$$\Delta_{DESI} = (M_\Omega - M_{\Xi^*}) - (M_{\Sigma^*} - M_\Delta), \quad (3.5)$$

$$\Delta_{DESII} = \frac{1}{2}(M_{\Sigma^*} - M_\Delta) + \frac{1}{2}(M_\Omega - M_{\Xi^*}) - M_{\Xi^*} + M_{\Sigma^*}. \quad (3.6)$$

All three of these combinations of masses vanish exactly to leading order in the  $SU(3)$  symmetry terms,  $b_F$ ,  $b_D$  and  $m_\nu^2$ .

For the baryon masses listed in Table 2, we find  $\Delta_{GMO} = 6$  MeV and  $\Delta_{DESI} = -2$  MeV, to be compared with the empirical violations of 6.6 and  $-13.6$  MeV respectively. The discrepancy between our result and the DES I relation is largely due to the fact that our calculated  $\Omega$  mass is out by 10 MeV. The DES II relation, which has an empirical violation of  $-3$  MeV, is a rather poor test of the octet nature of the  $SU(3)$  breaking. Our one-loop self energies would satisfy it exactly if we used the simple bare splittings of Eq. (2.1) together with  $SU(3)$  symmetric meson-baryon couplings. As has been noted by Jenkins [18], it is also

satisfied exactly to order  $m_K^4$  in a chiral expansion. In a  $1/N_c$  expansion violations of this relation first appear at order  $m_s^3/N_c^2$  and so are highly suppressed [28]. In our results the violation is very small, less than 0.1 MeV.

Thus despite the importance of higher-order terms which can transform under a variety of representations of  $SU(3)$ , the pattern of baryon masses remains close to that expected from purely octet symmetry breaking, a point first made by Jaffe [38] in the context of a chiral bag model. Hence the observed success of the GMO relation cannot be used to infer that baryon masses can be described using first-order perturbation theory in the current quark masses. For example, to first order in the current quark masses the  $SU(3)$ -breaking matrix element for the proton is given by

$$\frac{1}{3}(\overline{m} - m_s)\langle p|\bar{u}u + \bar{d}d - 2\bar{s}s|p\rangle \simeq M_\Lambda - M_\Xi = -202 \text{ MeV}. \quad (3.7)$$

The contribution of the non-strange quark masses to the proton mass is given by the  $\pi N$  sigma commutator,

$$\sigma_{\pi N} = \overline{m}\langle p|\bar{u}u + \bar{d}d|p\rangle \simeq 45 \text{ MeV}, \quad (3.8)$$

where the value quoted is from the analysis of  $\pi N$  scattering by Gasser *et al.* [39]. Combining Eqs. (3.7) and (3.8) leads to an estimate of the contribution of the strange quark mass to the nucleon mass that is surprisingly large [40]:

$$m_s\langle p|\bar{s}s|p\rangle \simeq \frac{1}{2} \left[ \frac{m_s}{\overline{m}}\sigma_{\pi N} - 3\frac{M_\Xi - M_\Lambda}{1 - \overline{m}/m_s} \right] \simeq 260 \text{ MeV}, \quad (3.9)$$

where the standard PCAC estimate of the ratio of quark masses [2],  $m_s/\overline{m} \simeq 25$ , has been used. Hence the use of first-order perturbation theory would suggest a large strange-quark content of nucleon.

Going beyond perturbation theory, the contributions of the quark masses to the nucleon mass can be determined by applying the Feynman-Hellmann theorem. The various contributions to the scalar quark densities from the meson clouds and from intermediate-state baryons are:

$$\overline{m}\langle N|\bar{u}u + \bar{d}d|N\rangle_\pi = m_\pi^2 \frac{\partial \Sigma_N}{\partial m_\pi^2} \quad (3.10)$$

$$\overline{m}\langle N|\bar{u}u + \bar{d}d|N\rangle_\eta = \frac{m_\pi^2}{3} \frac{\partial \Sigma_N}{\partial m_\eta^2} \quad (3.11)$$

$$\overline{m}\langle N|\bar{u}u + \bar{d}d|N\rangle_K = \frac{m_\pi^2}{2} \frac{\partial \Sigma_N}{\partial m_K^2} \quad (3.12)$$

$$m_s\langle N|\bar{s}s|N\rangle_K = \frac{(2m_K^2 - m_\pi^2)}{2} \frac{\partial \Sigma_N}{\partial m_K^2} \quad (3.13)$$

$$m_s\langle N|\bar{s}s|N\rangle_\eta = \frac{3(m_\eta^2 - m_\pi^2)}{4} \frac{\partial \Sigma_N}{\partial m_\eta^2} \quad (3.14)$$

$$m_s\langle N|\bar{s}s|N\rangle_B = b_F \frac{\partial \Sigma_N}{\partial b_F} + b_D \frac{\partial \Sigma_N}{\partial b_D} + b_{10} \frac{\partial \Sigma_N}{\partial b_{10}} \quad (3.15)$$

The values of these are listed in Table 3.

We find similar results to Eq. (3.9) if we keep only terms of first order in the symmetry-breaking. However the slow convergence of the expansion in  $m_K$  means that such large values for the strangeness content should not be taken seriously. Indeed to next order ( $m_K^3$ ) we find negative values for  $m_s\langle p|\bar{s}s|p\rangle$  (as in Ref. [21]) reflecting the alternating signs of the terms in the self-energy that can be seen in Figs. 1 and 2. With the full loop integrals, the energy denominators ensure that kaon and  $\eta$  contributions to the self-energy are small, and hence that the strange quark mass contributes only about 15 MeV to the nucleon mass. In contrast the pion cloud contributes significantly to the  $\pi N$  sigma commutator, adding about 20 MeV to the 15 – 20 MeV that the valence-quark core would provide in a bag or soliton model. Including meson cloud effects and treating SU(3)-breaking nonperturbatively, it is thus possible to have a large  $\sigma_{\pi N}$  without a large strangeness content in the nucleon, an observation that has been made in the context of various bag and soliton models [38,41,42].

#### IV. CONCLUSIONS

We have examined the self-energies of the octet and decuplet baryons within the framework of a generic cloudy-bag approach. These energies are calculated at one-loop level from intermediate states consisting of a virtual pseudoscalar meson and an octet or decuplet baryon. Meson-baryon form factors are used at the vertices, and these regulate the

loop integrals. In a chiral expansion of the self-energies in powers of the meson masses, the form factors can be thought of as providing model estimates for some of the higher-order counterterms that would be present in a complete ChPT treatment.

The chiral expansion converges rapidly for the pion loops with intermediate octet baryons, terms up to  $m_\pi^4$  being sufficient to accurately reproduce the full integral. For loops with intermediate decuplet baryons, the bare octet-decuplet splitting  $\delta$  provides a small energy denominator that could lead to very poor convergence of a chiral expansion in  $m_\pi$  alone. Provided that one keeps terms to all orders in  $m_\pi/\delta$  are kept, similar the contributions from pion-decuplet intermediate states show a similar convergence to those from pion-octet states. In contrast the expansion of kaon loops is much slower, with terms being needed up to order  $m_K^7$  at least. Our model for the higher-order counter terms of ChPT suggests that chiral expansions are unlikely to be of much use for kaon-baryon physics, in contrast to the situation for pions and baryons. This expectation is borne out by the lack of convergence found in recent ChPT calculations to order  $m_K^4$  [16].

A good description of the octet and decuplet baryon spectrum can be obtained in this approach, provided that we include bare splittings that go beyond those of a simple SU(6) quark model. Relations such as GMO and the decuplet equal spacing rules are very well satisfied by the calculated masses. Hence, despite the need for high-order terms in the chiral expansion, the pattern of splittings remains close to that produced by a purely octet SU(3)-breaking term. The importance of such terms means that the observed  $\pi N$  sigma commutator cannot be used to deduce a large strangeness content of the nucleon. In our treatment the pion cloud contributes nearly half of the observed sigma commutator, yet the strangeness content of the proton is very small.

We believe that our approach provides reasonable estimates of the higher-order terms in a chiral expansion. The results indicate that, while such an expansion converges well enough to be useful for pion-nucleon physics, this is not the case for kaons. Hence estimates of the strangeness content of the proton based on low-order terms of a chiral expansion are not reliable.

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## REFERENCES

- [1] S. Weinberg, *Physica* **96A** (1979) 327.
- [2] J. Gasser and H. Leutwyler, *Phys. Reports* **87** (1982) 77.
- [3] J. Gasser and H. Leutwyler, *Ann. Phys.* **158** (1984) 142.
- [4] J. F. Donoghue, E. Golowich and B. R. Holstein, *Dynamics of the standard model* (Cambridge University Press, Cambridge, 1992).
- [5] U.-G. Meissner, *Rep. Prog. Phys.* **56** (1993) 903.
- [6] V. Bernard, N. Kaiser and U.-G. Meissner, *Int. J. Mod. Phys.* **4** (1995) 193.
- [7] J. Gasser, M. E. Sainio and A. Svarc, *Nucl. Phys.* **B307** 779.
- [8] E. Eichten and B. Hill, *Phys. Lett.* **B234** (1990) 511;  
H. Georgi, *Phys. Lett.* **B240** (1990) 447.
- [9] E. Jenkins and A. V. Manohar, *Phys. Lett.* **B255** (1991) 558.
- [10] S. Th  berge, A. W. Thomas and G. A. Miller, *Phys. Rev.* **D22** (1980) 2838;  
A. W. Thomas, S. Th  berge and G. A. Miller, *Phys. Rev.* **D24** (1981) 216.
- [11] A. W. Thomas, *Adv. Nucl. Phys.* **13** (1984) 1.
- [12] A. G. Williams and L. R. Dodd, *Phys. Rev.* **D37** (1988) 1971.
- [13] J. A. McGovern, *Nucl. Phys.* **A533** (1991) 553.
- [14] L. R. Dodd and D. E. Driscoll, *Phys. Rev.* **C47** (1993) 1791.
- [15] M.-S. Bae and J. A. McGovern, *J. Phys. G: Nucl. Part. Phys.* **22** (1996) 199.
- [16] B. Borasoy and U.-G. Meissner, *Phys. Lett.* **B365** (1996) 285.
- [17] E. Jenkins and A. V. Manohar, *Phys. Lett.* **B259** (1991) 353.
- [18] E. Jenkins, *Nucl. Phys.* **B368** (1992) 190.

- [19] R. F. Lebed and M. A. Luty, Phys. Lett. **B329** (1994) 479.
- [20] E. Jenkins and A. V. Manohar, Phys. Lett. **B281** (1992) 336.
- [21] V. Bernard, N. Kaiser and U.-G. Meissner, Z. Phys. **C60** (1993) 111.
- [22] M. K. Banerjee and J. Milana, Phys. Rev. **D52** (1995) 6451.
- [23] T. H. R. Skyrme, Proc. Roy. Soc. **A260** (1961) 127; Nucl. Phys. **31** (1962) 556.
- [24] I. Zahed and G. E. Brown, Phys. Rep. **142** (1986) 481.
- [25] T. Meissner, E. Ruiz Arriola, A. Blotz and K. Goeke, Ruhr-University of Bochum preprint RUB-TPII-42/93 (1993), hep-ph/9401216;  
R. Alkofer, H. Reinhardt and H. Weigel, University of Tübingen preprint (1995), hep-ph/9501213.
- [26] T. D. Cohen and W. Broniowski, Phys. Lett. **B292** (1992) 5;  
T. D. Cohen, University of Maryland preprint 96-057, hep-ph/9512275.
- [27] R. Dashen, E. Jenkins and A. V. Manohar, Phys. Rev. **D49** (1994) 4713; Phys. Rev. **D51** (1995) 3697.
- [28] E. Jenkins and R. F. Lebed, Phys. Rev. **D52** (1995) 282.
- [29] E. Jenkins, University of California at San Diego preprint UCSD/PTH 95-17 (1995), hep-ph/9509413.
- [30] M. Gell-Mann, Phys. Rev. **125** (1962) 1067;  
S. Okubo, Prog. Theor. Phys. **27** (1962) 949.
- [31] M. A. Morgan, G. A. Miller and A. W. Thomas, Phys. Rev. **D33** (1986) 817.
- [32] M.-S. Bae, University of Manchester M.Sc. thesis (1993).
- [33] R. G. Leech and M. C. Birse, J. Phys. G: Nucl. Part. Phys. **18** (1992) 785.



- [34] S. Wolfram, *Mathematica* (Addison-Wesley, Redwood City, 1991).
- [35] E. Jenkins, M. Luke, A. V. Manohar and M. J. Savage, Phys. Lett. **B302** (1993) 482.
- [36] I. Jameson, A. A. Rawlinson and A. W. Thomas, Aust. J. Phys. **47** (1994) 45.
- [37] R. L. Jaffe and A. V. Manohar, Nucl. Phys. **B337** (1990) 509;  
F. E. Close and R. G. Roberts, Phys. Lett. B316 (1993) 165;  
P. G. Ratcliffe, University of Milan preprint, hep-ph/9506288.
- [38] R. L. Jaffe, Phys. Rev. **D21** (1980) 3215; Nucl. Phys. **A478** (1988) 3c.
- [39] J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. **B253** (1991) 252, 260.
- [40] T. P. Cheng, Phys. Rev. **D13** (1976) 2161.
- [41] M. C. Birse and J. A. McGovern, Phys. Lett. **B292** (1992) 242.;  
I. Jameson, A. W. Thomas and G. Chanfray, J. Phys. G: Nucl. Part. Phys. **18** (1992)  
L159.
- [42] J.-P. Blaizot, M. Rho and N. N. Scoccola, Phys. Lett. **B209** (1988) 27;  
J. A. McGovern and M. C. Birse, *ibid.* 163.

TABLES

$A \ B$	$N$	$\Sigma$	$\Lambda$	$\Xi$	$\Delta$	$\Sigma^*$	$\Xi^*$	$\Omega$
$N$	$9(F + D)^2$ $9F^2 - 6DF + D^2$	$\underline{9(F - D)^2}$	$\underline{9F^2 + 6DF + D^2}$		32	<u>8</u>		
$\Sigma$	$\underline{6(F - D)^2}$	$24F^2$ $4D^2$	$4D^2$	$\underline{6(F + D)^2}$	$\frac{64}{3}$	$\frac{16}{3}$ 8	$\frac{16}{3}$	
$\Lambda$	$18F^2 + 12DF + 2D^2$	$\underline{12D^2}$	$\underline{4D^2}$	$18F^2 - 12DF + 2D^2$		24	<u>16</u>	
$\Xi$		$9(F + D)^2$	$9F^2 - 6DF + D^2$	$\underline{9(F - D)^2}$ $\underline{9F^2 + 6DF + D^2}$		<u>8</u>	8 8	<u>16</u>
$\Delta$	8	<u>8</u>			25 5	<u>10</u>		
$\Sigma^*$	$\frac{8}{3}$	$\frac{8}{3}$ <u>4</u>	4	$\frac{8}{3}$	$\frac{40}{3}$	$\frac{40}{3}$	$\frac{40}{3}$	
$\Xi^*$		<u>4</u>	4	<u>4</u> <u>4</u>		20	5 <u>5</u>	<u>10</u>
$\Omega$				<u>16</u>			20	<u>20</u>

TABLE I. Squares of the SU(3) coefficients  $f^{AB\nu}$  appearing in the one-loop self energy of baryon  $A$  involving an intermediate baryon state  $B$  and a meson. In the strangeness-conserving entries the first line gives the pion coefficient, the second the  $\eta$  one. The underlined entries are those for which  $f_{AB\nu}$  is negative. In an SU(6) quark model the two octet-octet couplings are given by  $D = 1$  and  $F = \frac{2}{3}$ .

	N	$\Lambda$	$\Sigma$	$\Xi$	$\Delta$	$\Sigma^*$	$\Xi^*$	$\Omega$
SU(6)	939	1097	1193	1256	1232	1385	1535	1685
Full	939	1115	1194	1318	1232	1384	1535	1684
Expt.	939	1116	1193	1318	1232	1385	1533	1672

TABLE II. Baryon masses in MeV. The first line shows results of a calculation using the bare mass splittings and meson-baryon couplings of an SU(6) quark model. The second shows results of a calculation with  $F/D = 0.58$  and with the general bare splittings of (3.3) adjusted to fit the observed  $\Sigma$ - $\Lambda$  average,  $\Xi$ ,  $\Delta$  and  $\Xi^*$  energies.

$\overline{m}\langle u\bar{u} + d\bar{d}\rangle_\pi$	$\overline{m}\langle u\bar{u} + d\bar{d}\rangle_K$	$\overline{m}\langle u\bar{u} + d\bar{d}\rangle_\eta$	$m_s\langle s\bar{s}\rangle_K$	$m_s\langle s\bar{s}\rangle_\eta$	$m_s\langle s\bar{s}\rangle_B$
22.7	0.7	-0.02	16.7	0.5	6.5

TABLE III. Contributions (in MeV) to the nonstrange and strange quark scalar densities from the meson cloud and the intermediate baryons.

## FIGURES

FIG. 1. Dependence on meson mass of the octet baryon self-energy with meson-octet-baryon intermediate states.

FIG. 2. Dependence on meson mass of the octet baryon self-energy with meson-decuplet-baryon intermediate states.

Fig. 1

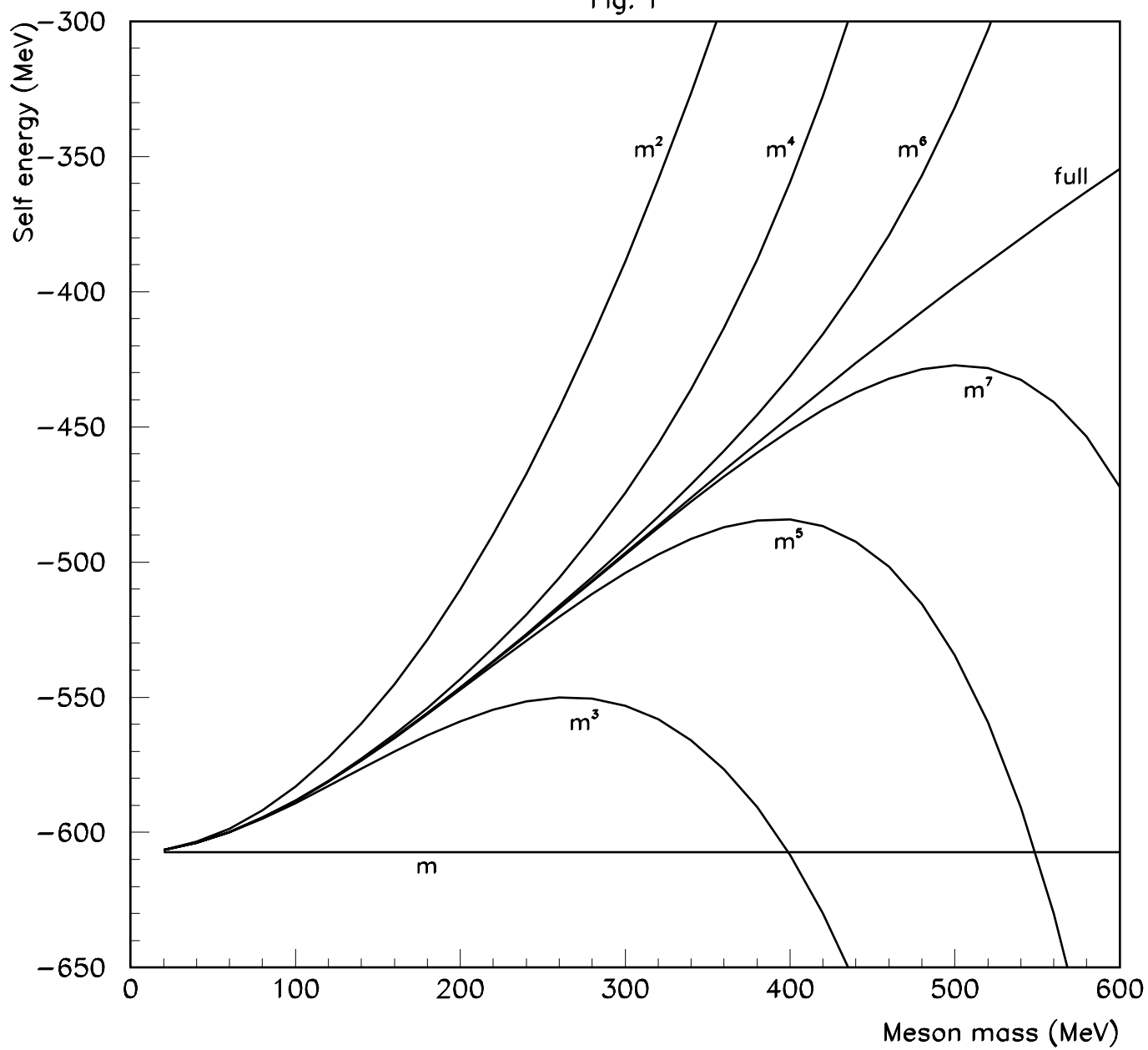


Fig. 2

